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A NUMERICAL CALCULATION OF NONLINEAR TRANSIENT HEAT CONDUCTION IN THE FUEL ELEMENTS OF A NUCLEAR REACTOR

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Abstract—A variational method for the solution of the transient heat transfer problem in nuclear reactor elements is proposed. The starting point is Lebon–Lambermont's variational principle which has proved to be particularly adequate in treating heat conduction problems. Kantorovitch's partial integration procedure is used. After selection of a trial function involving two unknown time dependent parameters, the temperature distributions in the fuel pin and in the cladding are calculated. In particular, the consequences of a shutdown in a nuclear reactor are analyzed. The method is general in that it can be applied whatever the nature of the boundary conditions, linear or nonlinear, and the dependence of the thermal properties on the temperature.

NOMENCLATURE

a, ratio of outer to inner radii in a hollow cylinder;
c, heat capacity [$\mathrm{J\,kg^{-1}\,K^{-1}}$];
f_v, Helmholtz free energy per unit volume [$\mathrm{J\,m^{-3}}$];
h, convective heat transfer coefficient [$\mathrm{W\,m^{-2}\,K^{-1}}$];
k, heat conductivity [$\mathrm{W\,m^{-1}\,K^{-1}}$];
L_j, integral defined by equation (4.9);
M_j, integral defined by equation (4.10);
$\mathbf{n}$, unit normal;
N_2, integral defined by equation (4.11);
q, non-dimensional parameter in the trial functions;
$\mathbf{q}$, heat flux [$\mathrm{W\,m^{-2}}$];
r, radial coordinate in a cylindrical system [m];
R_0, radius of the central hole in the fuel pin [m];
R_f, external radius of the fuel pin;
R_g, inner radius of the clad;
R_w, outer radius of the clad;
s_v, entropy per unit volume [$\mathrm{J\,m^{-3}\,K^{-1}}$];
t, time [s];
T, temperature [K];
u_v, internal energy per unit volume [$\mathrm{J\,m^{-3}}$];
V, volume [$\mathrm{m^3}$];
W, heat source per unit volume [$\mathrm{W\,m^{-3}}$];
Z, non-dimensional radial coordinate.

Greek symbols

α, thermal expansion coefficient;
δ, symbol of variation;
Λ, non-dimensional thickness of a hollow cylinder;
ξ, porosity [%];
ρ, density [$\mathrm{kg\,m^{-3}}$];
σ, entropy production per unit volume [$\mathrm{W\,m^{-3}\,K^{-1}}$];
θ, non-dimensional temperature;
τ, non-dimensional time.

Subscripts

C, coolant;
F, fuel;
G, clad;
ref, reference quantity;
*, non-dimensional quantity;
0, initial quantity;
', time derivation.

1. INTRODUCTION

THE KNOWLEDGE of transient temperature profiles plays an important role in the thermohydraulic design of nuclear reactors. In particular, in view of simulating accidents, the temperature response of a fuel element to a variation of heat sources must be known. The problem is highly nonlinear: nonlinearity arises by requiring that the thermal properties, like heat conductivity, heat capacity and density, as well as the boundary conditions are temperature dependent.

The purpose of this work is to provide an approximate solution with a minimum computing time. A variational method has been selected. The main advantages of variational techniques are their short computational times, their simplicity and the fact that the solutions are given in analytic forms.

The method developed in this paper rests on Lebon–Lambermont's variational principle [1, 8, 9, 11, 12]. Like many other variational criteria of continuum physics [2–7], it has to be classified as a restricted principle [10]. It means that some quantities are to be held constant during the variational procedure.

After giving a description of the problem to be solved (Section 2), the variational criterion is briefly presented (Section 3). The explicit expression of the functional to be varied is established.

Using Kantorovitch's partial integration method, an approximate solution is provided in Section 4. Numerical results are presented and discussed in Section 5.

2. DESCRIPTION OF THE SYSTEM

The purpose of this paper is to predict the transitory temperature profile in the fuel element represented in Fig. 1. The element consists of a hollow cylinder of uranium oxide encased in a cylindrical metallic cladding. Because the initial time is taken after a certain irradiation time, a hole has been formed at the centrum of the pin due to irreversible structural changes in the fuel material. The fuel pin and the cladding are separated by a small gap. Heat is generated in the fuel pin by a source which is a function of space and time. The outer surface of the clad is cooled by a refrigerant axial flow.

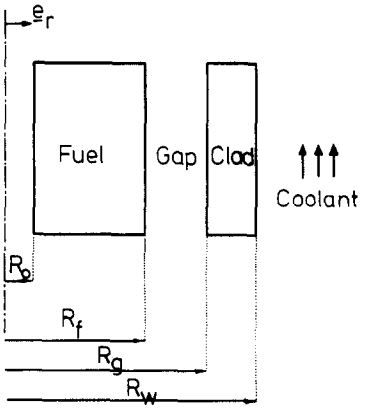

FIG. 1. Geometrical representation of a fuel element.

The actual problem is a moving boundary problem since the fuel–clad gap changes with time. In the present work, this effect is modelled by assigning a time dependence to the gas gap heat transfer coefficient and by keeping the distance fuel–clad constant.

Axial and azimuthal effects are ignored so that the problem is one-dimensional in space.

In cylindrical coordinates, the heat conduction equation is given by:

$$\rho_i(T_i)c_i(T_i)\frac{\partial}{\partial t}T_i = k_i(T_i)\frac{\partial^2 T_i}{\partial r^2} + \frac{k_i(T_i)}{r}\frac{\partial T_i}{\partial r} + \frac{\partial k_i(T_i)}{\partial r}\frac{\partial T_i}{\partial r} + W_i(r,t), \quad i = F, G. \tag{2.1}$$

Subscript $i = F, G$ refers to the fuel ($i = F$) and the clad ($i = G$) respectively.

When this equation is applied to the clad, the source term must be cancelled, since metallic phase changes and chemical reactions with the coolant are not taken into account. In the above equation, T denotes the absolute temperature, ρ the density, c the heat capacity, k the heat conductivity and W the heat source term.

The appropriate boundary conditions are

$$\frac{\partial T_F}{\partial r} = 0, \quad \text{at } r = R_0 \tag{2.2}$$

$$R_f \mathbf{q}_F \cdot \mathbf{n} = R_f h_{FG}(t)[T_F - T_G(R_g)], \quad \text{at } r = R_f \tag{2.3}$$

$$-R_g \mathbf{q}_G \cdot \mathbf{n} = R_f h_{FG}(t)[T_G - T_F(R_f)], \quad \text{at } r = R_g \tag{2.4}$$

$$R_w \mathbf{q}_G \cdot \mathbf{n} = R_w h_{GC}(t)(T_G - T_C), \quad \text{at } r = R_w. \tag{2.5}$$

$\mathbf{n}$ denotes a unit normal vector directed away from the axis of symmetry, h_{fG} and h_{GC} are the gap and coolant heat transfer coefficients, supposed to vary with time only; $\mathbf{q}_f$ and $\mathbf{q}_G$ are the heat flux vectors related to the temperature gradient by Fourier's law,

$$\mathbf{q} = -k\frac{\partial T}{\partial r}\mathbf{e}_r. \tag{2.6}$$

$\mathbf{e}_r$ is the unit vector in the radial direction. In formulating equations (2.3) and (2.4), nonlinear radiative boundary terms have been omitted. This contribution has been taken into account by Rafalski and Szczurek [13, 14] who used Biot's variational principle to evaluate the temperature field in a fuel pin with constant thermal properties.

The boundary conditions (2.2)–(2.5) must be complemented by the initial conditions,

$$T_F(0, r) = T_{0F}(r), \quad T_G(0, r) = T_{0G}(r), \tag{2.7}$$

where T_{0F} and T_{0G} are given functions of the radial distance.

It is convenient to formulate the above equations in terms of the following non-dimensional quantities:

Radial distance: $$Z = \frac{r}{R_f}, \quad (R_f = 0.2121 \text{ cm})$$

time: $$\tau = \frac{k_{\text{ref}}}{R_f^2 \rho_{\text{ref}} c_{\text{ref}}} t$$

temperature: $$\theta = \frac{T}{T_{\text{ref}}}$$

density: $$\rho^* = \frac{\rho}{\rho_{\text{ref}}}, \quad (\rho_{\text{ref}} = 10.983\,\text{g cm}^{-3})$$

heat capacity: $$c^* = \frac{c}{c_{\text{ref}}}$$

heat conductivity: $$k^* = \frac{k}{k_{\text{ref}}}$$

heat transfer coefficient: $$h^* = \frac{R_f}{k_{\text{ref}}} h$$

heat source term: $$W^* = \frac{R_f^2}{k_{\text{ref}} T_{\text{ref}}} W.$$

As reference values, one has selected the values of the thermal properties of the fuel at a given temperature, namely 1000°C. In non-dimensional notation, expressions (2.1)–(2.5) become

$$\rho_i^* c_i^* \frac{\partial \theta_i}{\partial t} = k_i^* \frac{\partial^2 \theta_i}{\partial Z^2} + \frac{k_i^*}{Z} \frac{\partial \theta_i}{\partial Z} + \frac{\partial k_i^*}{\partial Z} \frac{\partial \theta_i}{\partial Z} + W_i^*, \tag{2.8}$$

with

$$\frac{\partial \theta_F^*}{\partial Z} = 0, \quad \text{at } Z = Z_0 \tag{2.9}$$

$$-k_F^* \frac{\partial \theta_F}{\partial Z} = h_{FG}^* [\theta_F - \theta_G(Z_g)], \quad \text{at } Z = 1 \tag{2.10}$$

$$Z_g k_G^* \frac{\partial \theta_G}{\partial Z} = h_{FG}^* [\theta_G - \theta_F(1)], \quad \text{at } Z = Z_g \tag{2.11}$$

$$-k_G^* \frac{\partial \theta_G}{\partial Z} = h_{GC}^* (\theta_G - \theta_C), \quad \text{at } Z = Z_w. \tag{2.12}$$

The method presented in the next sections is general and applies whatever the dependence of the thermal properties with respect to θ, Z and τ.

We shall use the following correlations, which are believed to be a realistic description of the properties of the fuel elements.

(a) *Fuel properties.*

Density [15]: $$\rho_F^* = \rho_{\text{th}}^* [1 + \alpha_F(\theta)\theta]^{-3}. \tag{2.13}$$

ρ_{th} is the theoretical density taken to be equal to the reference density, whence $\rho_{\text{th}}^* = 1$; α_F is the thermal expansion coefficient.

Heat capacity [16]:

$$c_F^* = 0.58179 + 1.00264\theta - 0.87823\theta^2 + 0.29324\theta^3. \tag{2.14}$$

Heat conductivity: $k_F^* = k_0^*(\theta) f(\xi)$

$$k_0^* = \frac{1.12829}{\theta + 0.1016} + 0.04747\theta^3 - 0.07172 \tag{2.15}$$

$$f(\xi) = 1 - 1.029\xi - 0.32\,10^{-3}\xi^2 - 0.401\,10^{-4}\xi^3 + 0.158\,10^{-5}\xi^4. \tag{2.16}$$

k_0^* is the thermal conductivity of a 100% dense fuel, expression (2.15) was proposed by Ogawa *et al.* [17]; ξ is the porosity (in %) assumed to be constant in time but varying radially according to a law $\xi = \xi(Z)$ obtained experimentally; expression (2.16) for $f(\xi)$ was proposed by Biancheria [18].

Internal heat supply:

$$W_F^*(Z, \tau) = g(Z) W(\tau), \tag{2.17}$$

where $g(Z)$ is a given function of Z, and the dependence of $W(\tau)$ with the time may be chosen in an arbitrary way. Two cases will be investigated:

(1):
$$W(\tau) = 1 - 5.032\tau, \quad \text{for } \tau < 0.169 (t < 1\,\text{s}) \tag{2.18}$$
$$= 0.15, \quad \text{for } \tau \geqslant 0.169 (t \geqslant 1\,\text{s}) \tag{2.19}$$

(2):
$$W(\tau) = 1 - 59.17\tau, \quad \text{for } \tau < 0.0169 (t < 0.1\,\text{s}) \tag{2.18'}$$
$$= 0, \quad \text{for } \tau \geqslant 0.0169 (t \geqslant 0.1\,\text{s}) \tag{2.19'}$$

Of course, our method is completely independent of the dependence of $W^*(Z, \tau)$ with respect to Z and τ.

Gap heat transfer coefficient, assumed to vary linearly with time according to the law, is:

$$h^*_{fG} = 4.29 - 20.90\tau, \quad \text{for } \tau < 0.169 (t < 1\,\text{s}) \tag{2.20}$$
$$= 0.76, \quad \text{for } \tau \geqslant 0.169 (t \geqslant 1\,\text{s}). \tag{2.21}$$

Like (2.18) and (2.19), these laws are arbitrary. Initial temperature distribution $\theta_{0f}(Z)$: given. The values of $\theta_{0f}(Z)$, $g(Z)$ and $f(\xi)$ are collected in Table 1.

Table 1. Experimentally given values of some parameters in the fuel

Z	θ_0	$g(Z)$	$f(\xi)$
1.0000	1.01338		0.927
		5.6593	
0.9500	1.10745		0.9293
		5.1081	
0.8967	1.20311		0.9183
		4.5470	
0.8397	1.27242		0.8969
		3.9689	
0.7779	1.37203		0.8339
		3.2982	
0.7053	1.4962		0.8348
		2.9204	
0.6233	1.61298		0.9794
		3.003	
0.5431	1.6808		0.9794
		2.6312	
0.4484	1.77451		0.9794
		2.2908	
0.3263	1.83721		0.9794
		1.9766	
0.108	1.88731		0.9794

(b) *Clad properties.*

Density:
$$\rho^*_G = \frac{0.72384}{(1 + 2.1641\,10^{-2}\theta)^3}. \tag{2.22}$$

Heat capacity:
$$c^*_G = 1.57806 + 0.36417(\theta - 0.21445) + 4.5837(\theta - 0.58366)^3. \tag{2.23}$$

Heat conductivity:
$$k^*_G = 3.35102 + 6.4321\theta. \tag{2.24}$$

Internal heat supply:
$$W^*_G = 0. \tag{2.25}$$

The coolant heat transfer coefficient, approximated by a law independent of the coolant flow Reynolds number, is:

$$h^*_{GC} = 114.1 - 585.37\tau, \quad \text{for } \tau < 0.169 (t < 1\,\text{s}) \tag{2.26}$$
$$h^*_{GC} = 15.21, \quad \text{for } \tau \geqslant 0.169 (t \geqslant 1\,\text{s}). \tag{2.27}$$

The temperature of the coolant is supposed to be uniform, radially and axially along the cladding but to increase linearly with the time up to one second:

$$\theta_c = 0.533 + 0.079\tau, \quad \text{for } \tau < 0.169 (t < 1\,\text{s}) \tag{2.28}$$
$$= 0.5463, \quad \text{for } \tau \geqslant 0.169 (t \geqslant 1\,\text{s}). \tag{2.29}$$

Initial temperature distribution, assumed to vary linearly with the radial distance as

$$\theta_{0G}(Z) = -0.0447\frac{Z-Z_g}{Z_w-Z_g}+0.5915. \tag{2.30}$$

3. A VARIATIONAL PROCEDURE

The variational method rests on Lebon–Lambermont's variational criterion.

When deformations are neglected, it is well known [19] that the entropy s of a heated body is a function of the internal energy u only. Referring all quantities per unit volume, one has:

$$s_v = s_v(u_v). \tag{3.1}$$

The index v means that the corresponding quantity is measured per unit volume.

When the temperature is prescribed everywhere at the surface of the body (Dirichlet's condition), Lebon–Lambermont's principle can be written as

$$\delta_t \int_V \int_\tau L\,\mathrm{d}V\,\mathrm{d}t = 0, \tag{3.2}$$

with

$$L = T^2\{s_v[T^{-1}]+\tfrac{1}{2}\sigma\}. \tag{3.3}$$

The quantity $s_v[T^{-1}]$ is the Legendre transform of s_v with respect to T^{-1}:

$$s_v[T^{-1}] = s_v - T^{-1}u_v, \tag{3.4}$$

while σ is the entropy production generated per unit volume:

$$\sigma = \frac{k(T)}{T^2}(\mathrm{grad}\ T)^2. \tag{3.5}$$

An upper dot means derivation with respect to the time, V denotes the volume of the body, t the time duration of the process while index t affecting the variation symbol δ means that T must be frozen during variation. Making use of the state equation

$$u_v = \int_0^T \rho(\alpha)c(\alpha)\,\mathrm{d}\alpha, \tag{3.6}$$

the principle (3.2) becomes:

$$\delta_t \int_V \int_0^t \left[T\int_0^T \rho(\alpha)c(\alpha)\,\mathrm{d}\alpha + \tfrac{1}{2}k(T)(\mathrm{grad}\ T)^2\right]\mathrm{d}V\,\mathrm{d}t = 0. \tag{3.7}$$

If k is temperature dependent, it must also be kept constant during the variation.

The variational equations describing the temperature distribution in both the fuel and the clad are a little more complicated than (3.7) due to the presence of an internal source term and radiation-type boundary conditions. In non-dimensional notation, the principle for the fuel is given by:

$$\delta_\tau \int_0^\tau \int_{Z_{in}}^1 \left\{\theta_F \int_0^{\theta_F} \rho_F^*(\alpha)c_F^*(\alpha)\,\mathrm{d}\alpha + \tfrac{1}{2}k_F^*(\theta_F)\left(\frac{\partial\theta_F}{\partial Z}\right)^2 - \int_0^{\theta_F} W^*(\alpha)\,\mathrm{d}\alpha\right\} Z\,\mathrm{d}Z\,\mathrm{d}\tau$$
$$+\delta\int_0^\tau \tfrac{1}{2}h_{FG}^*[\theta_F-\theta_G(Z_g)]^2\,\mathrm{d}\tau|_{Z=1} = 0. \tag{3.8}$$

For the clad, one has

$$\delta_\tau \int_0^\tau \int_{Z_{in}}^{Z_w} \left\{\theta_G \int_0^{\theta_G} \rho_G^*(\alpha)c_G^*(\alpha)\,\mathrm{d}\alpha + \tfrac{1}{2}k_G^*(\theta_G)\left(\frac{\partial\theta_G}{\partial Z}\right)^2\right\} Z\,\mathrm{d}Z\,\mathrm{d}\tau$$
$$+\tfrac{1}{2}Z_w\delta\int_0^\tau h_{GC}^*(\theta_G-\theta_C)^2\,\mathrm{d}\tau|_{Z=Z_w} + \tfrac{1}{2}\delta\int_0^\tau h_{FG}^*[\theta_G-\theta_F(1)]^2\,\mathrm{d}\tau|_{Z=Z_{in}} = 0. \tag{3.9}$$

In these expressions, the heat transfer coefficients may depend on the variables Z and τ but not on the temperature. By varying with respect to θ_F, one recovers from (3.8) the heat equation for the fuel and the appropriate boundary conditions (2.9) and (2.10). In the same way, by varying with respect to θ_G in (3.9), one derives that the necessary conditions for (3.9) to hold true are that equations (2.8) as well as (2.11) and (2.12) are satisfied.

We shall seek approximate solutions for the temperature profiles in both the fuel and the cladding by using Kantorovitch's partial integration method. It consists of separating the variables Z and τ in the trial function which is given in the general form:

$$\theta_i(Z,\tau)=\sum_{n=1}^{M} f_i^{(n)}(Z)q_i^{(n)}(\tau),\quad i=F,G. \tag{3.10}$$

$f^{(n)}(Z)$ are *a priori* given functions of the space varialbe and $q^{(n)}(\tau)$ are unknown functions. The latter are obtained as solutions of the Euler–Lagrange equations of the problem. In the present case, they take the form of ordinary differential equations of the first order.

4. TEMPERATURE FIELD IN THE FUEL ELEMENTS

By application of the variational operator δ respectively to the integrands of (3.8) and (3.9), recalling that $\dot{\theta}$ and $k^*(\theta)$ are not to be varied, one obtains:

For the fuel pin:

$$\int_0^{\tau}\int_{Z_i}^{1}\left[\rho^*c^*\dot{\theta}\delta\theta+k^*\frac{\partial\theta}{\partial Z}\delta\left(\frac{\partial\theta}{\partial Z}\right)-W^*\delta\theta\right]Z\,\mathrm{d}Z\,\mathrm{d}\tau+\int_0^{\tau}h^*_{FG}[\theta-\theta_G(Z_g)]\delta\theta\,\mathrm{d}\tau|_{Z=1}=0; \tag{4.1a}$$

For the cladding:

$$\int_0^{\tau}\int_{Z_w}^{Z_i}\left[\rho^*c^*\dot{\theta}\delta\theta+k^*\frac{\partial\theta}{\partial Z}\delta\left(\frac{\partial\theta}{\partial Z}\right)\right]Z\,\mathrm{d}Z\,\mathrm{d}\tau+Z_w\int_0^{\tau}h^*_{GC}(\theta-\theta_C)\delta\theta\,\mathrm{d}\tau|_{Z=Z_i}+\int_0^{\tau}h^*_{FG}[\theta-\theta_I(1)]\delta\theta\,\mathrm{d}\tau|_{Z=Z_w}=0. \tag{4.1b}$$

For simplicity, index τ affecting symbol δ has been omitted. Since no confusion is possible, indices F and G have also been omitted.

4.1. *Temperature field in the fuel*

We take for the temperature field θ a trial function of the form:

$$\theta(Z,\tau)=\frac{1}{\Delta^2}(Z-Z_0)^2(q_1-q_2)+q_2,\quad(\Delta=1-Z_0), \tag{4.2}$$

from which,

$$\delta\theta(Z,\tau)=\frac{1}{\Delta^2}(Z-Z_0)^2\delta q_1+\left[1-\frac{1}{\Delta^2}(Z-Z_0)^2\right]\delta q_2. \tag{4.3}$$

Physically, q_1 and q_2 represent the temperatures of the outer and inner faces respectively.

Both parameters q_1 and q_2 are unknown functions of time, their initial values are determined with the help of the least square principle:

$$\delta\int_{Z_i}^{1}\left[\theta(Z,0)-\theta_{0F}(Z)\right]^2Z\,\mathrm{d}Z=0. \tag{4.4}$$

$\theta(Z,0)$ is the expression (4.2) wherein q_1 and q_2 have been replaced by their initial value: one finds, with the experimentally given $\theta_{0I}(Z)$,

$$q_1(0)=1.0121,\quad q_2(0)=1.8898. \tag{4.5}$$

Substitution of (4.2) and (4.3) in (4.1a) produces a linear form

$$Q_1(Z,\tau)\delta q_1+Q_2(Z,\tau)\delta q_2=0. \tag{4.6}$$

Since δq_1 and δq_2 are arbitrary, the coefficients Q_1 and Q_2 must identically vanish.

They are respectively given by:

$$(\dot{q}_1-\dot{q}_2)\frac{1}{\Delta^4}\int_{Z_i}^{1}\rho^*c^*(Z-Z_0)^4Z\,\mathrm{d}Z+\frac{\dot{q}_2}{\Delta^2}\int_{Z_i}^{1}\rho^*c^*(Z-Z_0)^2Z\,\mathrm{d}Z$$
$$-\frac{1}{\Delta^2}\int_{Z_i}^{1}W^*(Z-Z_0)^2Z\,\mathrm{d}Z+\frac{4}{\Delta^4}(q_1-q_2)\int_{Z_i}^{1}k^*(Z-Z_0)^2Z\,\mathrm{d}Z+h^*_{FG}[q_1-\theta_G(Z_g)]=0, \tag{4.7}$$

and

$$\frac{(\dot{q}_1-\dot{q}_2)}{\Delta^2}\left\{\int_{Z_i}^{1}\rho^*c^*(Z-Z_0)^2\left[1-\frac{1}{\Delta^2}(Z-Z_0)^2\right]Z\,\mathrm{d}Z+\dot{q}_2\left\{\int_{Z_i}^{1}\rho^*c^*\left[1-\frac{1}{\Delta^2}(Z-Z_0)^2\right]Z\,\mathrm{d}Z\right\}\right.$$
$$\left.-\int_{Z_i}^{1}W^*\left[1-\frac{1}{\Delta^2}(Z-Z_0)^2\right]Z\,\mathrm{d}Z-\frac{4}{\Delta^4}(q_1-q_2)\int_{Z_i}^{1}k^*(Z-Z_0)^2Z\,\mathrm{d}Z=0. \tag{4.8}$$

Setting:

$$L_j = \frac{1}{\Delta^{2j}} \int_{Z_0}^{1} \rho^* c^* (Z - Z_0)^{2j} Z \, dZ, \quad j = 0, 1, 2 \tag{4.9}$$

$$M_j = \frac{1}{\Delta^j} \int_{Z_0}^{1} W^* (Z - Z_0)^j Z \, dZ, \quad j = 0, 2 \tag{4.10}$$

$$N_2 = \frac{1}{\Delta^4} \int_{Z_0}^{1} k^* (Z - Z_0)^2 Z \, dZ, \tag{4.11}$$

expressions (4.7) and (4.8) may be written as

$$\begin{pmatrix} L_2 & L_1 - L_2 \\ L_1 - L_2 & L_2 - 2L_1 + L_0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} + \begin{pmatrix} 4N_2 + h_{FG}^* & -4N_2 \\ -4N_2 & 4N_2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} M_2 + h_{FG}^* \theta_G(Z) \\ M_0 - M_2 \end{pmatrix}, \tag{4.12}$$

or in a compact form,

$$\bar{\mathbf{A}} \cdot \dot{\mathbf{q}} + \bar{\mathbf{B}} \cdot \mathbf{q} = \mathbf{C}, \tag{4.13}$$

where $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ are symmetric matrices and $\mathbf{C}$ a vector. Solving with respect to $\dot{\mathbf{q}}$ yields:

$$\dot{\mathbf{q}} + \bar{\mathbf{D}} \cdot \mathbf{q} = \mathbf{d}, \tag{4.14}$$

where

$$\bar{\mathbf{D}} = \bar{\mathbf{A}}^{-1} \cdot \bar{\mathbf{B}}, \quad \mathbf{d} = \bar{\mathbf{A}}^{-1} \cdot \mathbf{C}. \tag{4.15}$$

In most problems, equation (4.14) is a nonlinear differential equation since generally $\bar{\mathbf{D}}$ and $\mathbf{d}$ depend themselves on $\mathbf{q}$ through the integrals L_j, M_j and N_2. In the particular case that the thermal properties as well as the source term are temperature-independent, equation (4.14) becomes linear and its solution is simply

$$\mathbf{q}(\tau) = e^{-\bar{D}\tau} \left| \int_0^{\tau} e^{\bar{D}\alpha} \mathbf{d}(\alpha) \, d\alpha + \mathbf{q}(0) \right|. \tag{4.16}$$

4.2. *Temperature field in the cladding*

The trial function is selected to be of the same form as for the fuel pin, i.e.

$$\theta(Z, \tau) = \frac{1}{\Delta^2} (Z_w - Z)^2 (q_3 - q_4) + q_4, \quad (\Delta = Z_w - Z_g), \tag{4.17}$$

q_4 and q_3 being respectively the outer and inner surface temperatures of the cladding.

Following the same procedure as in Section 4.1, one obtains the set of ordinary differential equations:

$$\begin{pmatrix} L_2 & L_1 - L_2 \\ L_1 - L_2 & L_2 - 2L_1 + L_0 \end{pmatrix} \begin{pmatrix} \dot{q}_3 \\ \dot{q}_4 \end{pmatrix} + \begin{pmatrix} 4N_2 + h_{FG}^* & -4N_2 \\ -4N_2 & 4N_2 + Z_w h_{GC}^* \end{pmatrix} \begin{pmatrix} q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} M_2 + h_{FG}^* \theta_F(1) \\ -M_2 + M_0 + Z_w^* h_{GC}^* \theta_C \end{pmatrix}. \tag{4.18}$$

The integrals L_j, M_j and N_2 are the same as integrals (4.9) to (4.11) except that the lower and upper bounds are now Z_g and Z_w respectively and $\Delta = Z_w - Z_g$ instead of $1 - Z_0$.

This system may be written in the same matricial form as (4.14) and its solution is still expressed by (4.16).

In a hot state, after a sufficient irradiation time, the fuel pin expands by dilatation and comes in contact with the cladding so that $Z_g = Z_f = 1$ [20]; Z_w is taken to be equal to 1.19.

The temperature profile in the cladding is strongly correlated to what happens in the fuel: as seen in (4.18), it depends on the value of $\theta_F(1)$, derived from the fuel analysis. In the same way, it appears from (4.12) that the fuel temperature distribution depends on the value of $\theta_G(Z_g)$. In that respect, the problem is a coupled one.

In a previous work [20], we completely uncoupled the problem by assuming that the temperature $\theta_G(Z_g)$ at the inner surface of the cladding and the heat flux $\mathbf{q}_G$ through this face are extrapolated at each time step.

These restrictions are relaxed in the present work.

4.3. *Test of the quality of the trial functions*

By appealing to a variational technique involving either "exact" or restricted variational principles, the crucial point is the selection of the most adequate trial function. Clearly, the better the choice of the trial function, the better the final result. To check the quality of the expansions (4.2) and (4.17) we compare with a similar linear problem whose exact solution is known.

It is the case for the problem of transient heat conduction in an infinite hollow cylinder ($1 < Z < a$) whose lateral surface are submitted to the following radiation-type boundary conditions:

$$\frac{\partial\theta}{\partial Z} = h_1^*(\theta-\theta_1), \quad \text{at} \quad Z = 1$$
$$a\frac{\partial\theta}{\partial Z} = -h_2^*(\theta-\theta_2), \quad \text{at} \quad Z = a. \tag{4.19}$$

h_1^* and h_2^* are the heat transfer coefficients at the surfaces $Z = 1$ and $Z = a$ respectively, θ_1 and θ_2 the constant temperatures of the surrounding media. For constant thermal properties and in absence of internal heat supply, the temperature profile is known and explicitly given in [21].

The variational and the exact solutions are compared in Table 2. The differences observed are of the order of 1%, which attests of the quality of the selected trial function. If the selected trial function had given poor results for this simplified problem, it is clear that it would have been inadequate for the more complicated fuel element problem.

5. NUMERICAL ANALYSIS

5.1. *The numerical scheme*

Instead of solving (4.14) by classical methods, like Runge–Kutta–Gill's technique, the nonlinear heat conduction problem has been approximated by a succession of linearized problems in order to reduce the computational time. If the macro-time interval $0-\tau$ is divided into several micro-steps $\Delta\tau$ sufficiently small, the thermal quantities and the heat source may be considered as constants, as well as $\bar{\mathbf{D}}$ and $\mathbf{d}$. Under these conditions, (4.16) furnishes an explicit finite difference scheme for the integration of (4.14) and can be written as

$$\mathbf{q}(\tau+\Delta\tau) = (\bar{\mathbf{I}}-e^{-\mathbf{D}\Delta\tau})\bar{\mathbf{D}}^{-1}\cdot\mathbf{d}+\mathbf{q}(\tau)e^{-\mathbf{D}\Delta\tau}, \tag{5.1}$$

where $\bar{\mathbf{I}}$ is the unity matrix.

To solve equation (5.1), one has the choice between three methods [22].

1. The forward explicit difference method: this consists of developing the matrix $\exp(-\Delta\tau\bar{\mathbf{D}})$ in a series expansion limited to the first order term in $\Delta\tau$, namely

$$\exp(-\Delta\tau\bar{\mathbf{D}}) = \bar{\mathbf{I}}-\Delta\tau\bar{\mathbf{D}}. \tag{5.2}$$

2. The backward implicit difference method: the matrix $\exp(-\Delta\tau\bar{\mathbf{D}})$ is approximated by $(\bar{\mathbf{I}}+\Delta\tau\bar{\mathbf{D}})^{-1}$.

3. The Crank–Nicolson implicit method which approximates the exponential term by:

$$\left(\bar{\mathbf{I}}+\frac{\Delta\tau}{2}\bar{\mathbf{D}}\right)^{-1}\cdot\left(\bar{\mathbf{I}}-\frac{\Delta\tau}{2}\bar{\mathbf{D}}\right). \tag{5.3}$$

It is interesting to compare the respective merits of these three methods.

While the first method is explicit, the two other ones are implicit, which implies the solution of a matrix problem.

A stability analysis based on Lyapounov's second method shows that the matrix approximations used in the backward difference and the Crank–Nicolson methods are unconditionally stable. On the other hand, the forward difference method is only stable for a time step obeying the following criterion:

$$0 \leqslant \Delta\tau \leqslant \min_{1\leqslant i\leqslant n}\left\{2\frac{\operatorname{Re}\lambda_i}{|\lambda_i|^2}\right\}; \tag{5.4}$$

the λ_i's are the eigenvalues of $\bar{\mathbf{D}}$, Re means their real part and $|\lambda_i|$ their norm.

Concerning the accuracy, the two first schemes are of the first order in $\Delta\tau$ while the Crank–Nicolson scheme is of the second order in $\Delta\tau$. The latter one is undoubtedly the most efficient from the numerical point of view. However, for a numerical code involving the shortest computing time, the explicit method is recommended.

Although the differential equation is given in matrix form, the numerical procedure based on an explicit method does not require the manipulation of matrices. Indeed, expression (5.1) reduces to a set of algebraic equations involving only exponential terms like $\exp(-\Delta\tau\lambda_i)$ coming from the diagonalization of the exponential matrices.

Moreover, since a piecewise linear approach is used to handle the non-linearities, the updating of $\mathbf{D}$ and $\mathbf{d}$ at each time step is required. Once more, the explicit scheme (5.1) is the most appropriate technique for achieving this objective.

For all these reasons, the forward explicit difference scheme has been retained in the present work. The limitation on the time-step $\Delta\tau$ imposed by (5.4) is not an important handicap with respect to implicit methods. Indeed, to attain a sufficient accuracy in an implicit scheme, the time step must be reduced until there is a negligible difference between two successive solutions. This needs several heavy programme executions. By using an explicit method, one avoids such additional calculations; the step size $\Delta\tau$ is selected as large as possible within the limits imposed by (5.4). Of course, to follow the fast transients of the present problem, $\Delta\tau$ will be chosen very short so that the restrictions prescribed by (5.4) are practically not constraining.

The time step $\Delta\tau$ is at most equal to twice the inverse of the greatest eigenvalue of $\bar{\mathbf{D}}$ in order to ensure stability of (5.1). In each micro-step $\Delta\tau$, the Kantorovitch procedure is repeated by taking as initial value for $\mathbf{q}$ the value computed at the end of the previous micro-step. In the coupled problem, the numerical calculations run as follows. The eigenvalues of $\bar{\mathbf{D}}$ are calculated in the fuel pin and in the cladding successively and the corresponding time

steps $\Delta\tau_F$ and $\Delta\tau_G$ in each of them are determined; $\Delta\tau_F$ is found to be of the order of $100\Delta\tau_G$.

The temperature distributions in the fuel pin, and in particular the outer surface temperature $\theta_F(1)$, are predicted by (5.1) at the end of a micro-step $\Delta\tau_F$. During each step $\Delta\tau_F$, the clad temperature distribution is then calculated with the help of (5.1) wherein $\Delta\tau = \Delta\tau_G$ until the value $\Delta\tau_F$ has been reached. The inner clad temperature $\theta_G(1)$ is then obtained. That value could be used to correct $\theta_F(1)$. The new value of $\theta_F(1)$ should allow itself the correction of $\theta_G(1)$. It has been verified that such a process does not increase significantly the precision so that a prediction-correction procedure is not necessary. The previous operations are repeated at each time step $\Delta\tau_F$ until the stationary state is reached in both the fuel pin and the cladding.

The method developed here presents the advantage of very short computational times. With an I.B.M. 370/158 computer, the C.P.U. time is of the order of 80 sec to reach the steady state. Of course, by using more unknowns, the dimension of the matrices $\bar{\mathbf{A}}, \bar{\mathbf{B}}, \mathbf{C}$ would increase and consequently the computing time.

5.2. *Results and discussion*

The fuel inner radius is taken to be equal to 10% of the external radius. The experimental source term $g(Z)$ and the porosity factor $f[\xi(Z)]$ are assumed to be fitted by the following Fourier expansions:

$$\left.\begin{matrix} g(Z) \\ f[\xi(Z)] \end{matrix}\right\} = \sum_{m=0}^{4} a_m \cos\left(m\frac{\pi}{1-Z_0}Z\right) + \sum_{m=1}^{4} b_m \sin\left(m\frac{\pi}{1-Z_0}Z\right).$$

The parameters a_m and b_m have been computed by a least square method.

The temperature profiles are presented in Figs. 2 and 3. During the decay in intensity of the heat source (from 100 to 15% of its initial value in 1 s in the first case, and from 100 to 0% in 0.1 s, in a second case), the following qualitative behaviour of the fuel element has been observed. The fuel temperature on the inner face of the central hole falls down continuously while it increases at the outer face; this phenomenon is observed during the decay of the source term. After that the source intensity is kept constant, the temperature decreases again at the outer face. This behaviour is due to the decay of the convective heat transfer coefficient h^*_{FG}.

The stationary state in the fuel is reached after 3.64 s in the first case, and after 2.4 s in the second

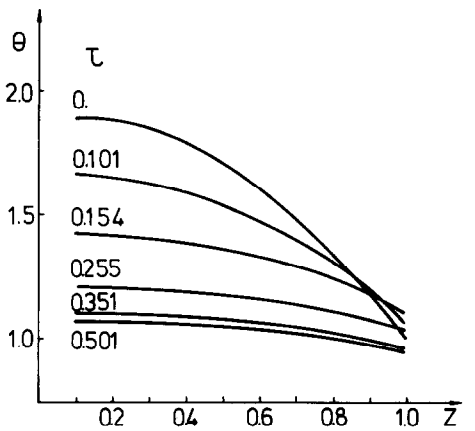

FIG. 2. Temperature profiles in the fuel pin as a function of the radial distance and the time. The heat source drops from 100 to 15% of its initial value in 1 s.

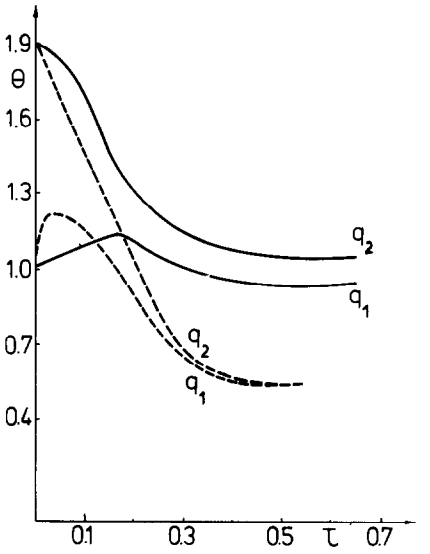

FIG. 3. Temperature profiles at the inner and outer faces of the fuel pin for two different behaviours of the heat source:
(1) decrease from 100% to 15% of its initial value in 1 s (continuous lines);
(2) decrease from 100% to zero in 0.1 s (dashed lines).

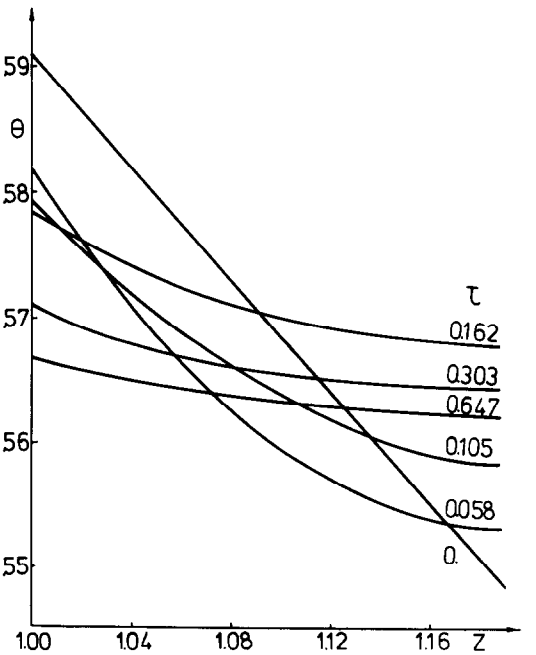

FIG. 4. Temperature profiles in the cladding as a function of the radial distance and the time. The heat source drops from 100 to 15% of its initial value in 1 s.

Table 2. Comparison between the exact θ_{theor} and the variational θ_{var} temperature profiles in a hollow cylinder (linear problem)

Z \ τ	0.1		0.2		0.3		0.35	
	θ_{theor}	θ_{var}	θ_{theor}	θ_{var}	θ_{theor}	θ_{var}	θ_{theor}	θ_{var}
1.039	2.686	2.649	2.761	2.793	2.765	2.806	2.765	2.807
1.059	2.664	2.631	2.737	2.777	2.740	2.790	2.740	2.791
1.079	2.644	2.611	2.715	2.761	2.719	2.774	2.719	2.775
1.099	2.627	2.599	2.696	2.745	2.699	2.758	2.700	2.759
1.119	2.611	2.586	2.680	2.730	2.683	2.743	2.683	2.743
1.139	2.599	2.574	2.666	2.715	2.669	2.727	2.669	2.728
1.159	2.588	2.564	2.654	2.700	2.657	2.713	2.657	2.713
1.179	2.580	2.555	2.646	2.687	2.649	2.698	2.649	2.699
1.199	2.575	2.548	2.639	2.673	2.642	2.684	2.642	2.684
1.219	2.571	2.542	2.636	2.659	2.699	2.670	2.639	2.670

Table 3. Variation in time of the following quantities: temperature at the inner (q_2) and outer (q_1) faces of the fuel pin, temperature at the inner (q_3) and outer (q_4) faces of the clad; inner heat flux through the clad ($\mathbf{q}_G \cdot \mathbf{n}$)

τ	t (s)	q_2 $\theta_F(Z_0)$	q_1 $\theta_F(1)$	q_3 $\theta_G(1)$	q_4 $\theta_G(Z_W)$	$\mathbf{q}_G \cdot \mathbf{n}$
0	0	1.890	1.012	0.592	0.547	1.804
0.058	0.343	1.814	1.054	0.582	0.533	1.461
0.105	0.619	1.654	1.086	0.579	0.559	1.069
0.162	0.960	1.404	1.133	0.578	0.568	0.502
0.198	1.170	1.316	1.105	0.577	0.569	0.400
0.303	1.796	1.145	0.996	0.571	0.565	0.322
0.401	2.373	1.070	0.948	0.568	0.562	0.288
0.494	2.924	1.050	0.935	0.568	0.562	0.279
0.616	3.650	1.046	0.932	0.567	0.562	0.277

Table 4a. Effects of the nonlinearities on the temperature profile in the fuel

	Relative error %	
Z \ τ	0.139	0.439
0.1	23.9	13.4
0.2	23.8	13.4
0.3	23.6	13.3
0.4	23.3	13.1
0.5	22.8	12.9
0.6	22.1	12.4
0.7	21.5	12.0
0.8	20.5	11.4
0.9	19.4	10.7
1.0	18.1	10.0

Table 4b.

Step number	Temperature dependent thermal properties τ	$\Delta\tau_F/\Delta\tau_G$	Constant thermal properties τ	$\Delta\tau_F/\Delta\tau_G$
1	0		0	
		54		24
2	0.018		0.020	
		98		24
3	0.057		0.062	
		98		24
4	0.096		0.104	
		98		24
5	0.0135		0.146	
		—		24
12	0.342		0.438	
		52		24
13	0.363		0.480	
		50		
14	0.382		Stationary state	
		—		
19	0.476			
		46		
20	0.495			
	Stationary state			

case. In fact, the stationary state is an equilibrium one characterized by a uniform temperature distribution.

The temperature field in the cladding is reported on Fig. 4 for the first case. There is a temperature jump at the interface between the fuel and the cladding due to the thermal resistance of the contact. The inner clad temperature decreases gradually to its stationary value while the outer temperature increases practically until the coolant temperature becomes constant (after 1 s); afterwards, it decreases to its stationary value. The temperature in the clad reaches its steady value faster than does the temperature in the fuel. Moreover, the former varies in a range of only 0.4% while the heat flux through the inner face of the clad drops appreciably (see Table 3).

To examine the effects of the nonlinearities, we have repeated the calculations by assuming that all the thermal properties are constant and equal to their value corresponding to the mean temperature $\theta = 1.58$. The fuel temperature distribution has been calculated at various instants of time for a drop of the heat source from its initial value to zero. The relative error with respect to the values calculated in the nonlinear case ranges between 10 and 24% (see Table 4a). Moreover, the ratio of the time steps $\Delta\tau_l/\Delta\tau_{l'}$ is now constant (see Table 4b) and equal to 24 while for the nonlinear problem it varies from step to step with a maximum value equal to 98. The stationary state is obtained after 20 steps in the nonlinear problem, after 13 steps in the linear case. All these results indicate clearly that the nonlinearities play an important role.

6. CONCLUSIONS

The transient temperature field in a fuel element has been computed by using Kantorovitch's variational technique. The method involves only two pairs of unknown parameters which are determined as solutions of four ordinary differential equations. The method is general in that it can be applied whatever the dependence of the thermal properties, the internal heat source terms and the boundary conditions with respect to the temperature or the space and time variables. The analysis has been restricted to a one-dimensional problem in space.

The extension to two or three dimensions should not raise fundamental difficulties but would complicate the procedure in that more involved trial functions must be selected. The same method can be easily extended for temperature dependent heat sources and heat transfer coefficients.

For the linear problem of heat conduction in an infinite hollow cylinder, the results of the variational method have been compared with the exact ones: an accuracy of 1% has been obtained. It is reasonable to expect the same order of accuracy for the problem of the heat transfer in the fuel element. This accuracy is more than sufficient for most situations encountered in nuclear engineering.

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CALCUL NUMERIQUE DE LA CONDUCTION THERMIQUE TRANSITOIRE ET NON-LINEAIRE DANS LES ELEMENTS COMBUSTIBLES DE REACTEUR NUCLEAIRE

Résumé—On propose une méthode variationnelle pour résoudre le problème du transfert thermique en régime transitoire dans les éléments d'un réacteur nucléaire. Le point de depart est le principe variationnel de Lebon–Lambermont, lequel s'est révélé être particulièrement efficace pour traiter les problèmes de conduction thermique. On utilise la méthode par intégration partielle de Kantorovitch. Après avoir sélectionné une fonction d'essai mettant en jeu deux paramètres inconnus mais fonction du

temps, on calcule la distribution de la température dans le crayon combustible et dans la gaine. On analyse en particulier les conséquences d'une chute de barres dans un réacteur nucléaire. La méthode est générale en ce sens qu'elle est susceptible d'être appliquée quelle que soit la nature des conditions aux limites, linéaires ou non-linéaires, et la forme de la dépendance des propriétés thermiques vis-à-vis de la température.

ЧИСЛЕННЫЙ РАСЧЁТ НЕЛИНЕЙНОГО НЕСТАЦИОНАРНОГО ПРОЦЕССА ТЕПЛОПРОВОДНОСТИ В ТВЭЛах ЯДЕРНОГО РЕАКТОРА

Аннотация — Предложен вариационный метод решения нестационарной задачи теплопроводности в ТВЭЛах ядерного реактора. За основу взят вариационный принцип Лебона–Ламбермона, справедливость которого была проверена при решении задач теплопроводности. Использован метод Канторовича интегрирования дифференциальных уравнений в частных производных. После выбора пробной функции, в которую входят два неизвестных, зависящих от времени, параметра, рассчитывается распределение температур в стержне и в оболочке элемента. В частности, проведен анализ последствий остановки ядерного реактора. Метод является общим, так как его можно использовать при любых граничных условиях, линейных или нелинейных, и при зависимости теплофизических свойств от температуры.

NUMERISCHE BERECHNUNG VON NICHTLINEARER INSTATIONÄRER WÄRMELEITUNG IN DEN BRENNELEMENTEN EINES KERNREAKTORS

Zusammenfassung—Es wird eine Variations-Methode zur Lösung des instationären Wärmetransportproblems in den Brennelementen eines Kernreaktors vorgeschlagen. Ausgangspunkt ist das Variations-Prinzip von Lebon-Lambermont, das sich zur Behandlung von Wärmeleitproblemen als besonders geeignet erwiesen hat. Das partielle Integrations-Verfahren von Kantorovitch wird angewandt. Nach Auswahl einer Ausgangs-Funktion mit zwei unbekannten zeitabhängigen Parametern wird die Temperaturverteilung im Brennstab und dessen Ummantelung berechnet. Insbesondere werden die Folgen einer Reaktorabschaltung analysiert. Die Methode ist allgemein und kann insofern für – lineare oder nichtlineare – beliebige Randbedingungen sowie für beliebig temperaturabhängige Stoffeigenschaften angewendet werden.